

## **Análisis de la instrumentalización e instrumentación que genera el uso de GeoGebra en la enseñanza tradicional de matemáticas**

### ***Analysis of the instrumentalization and instrumentation generated using GeoGebra in the traditional teaching of mathematics***

### ***Análise da instrumentalização e instrumentação gerada pelo uso do GeoGebra no ensino tradicional de matemática***

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## **Resumen**

La presente investigación analiza las consecuencias cognitivas que genera el uso del *software* GeoGebra como instrumento tecnológico para el aprendizaje de las matemáticas, específicamente en la unidad de aprendizaje de Geometría Analítica en el nivel medio superior (bachillerato). Para ello, se empleó la teoría de la instrumentación como marco teórico por ser pertinente para reconocer las limitaciones y posibilidades de un instrumento tecnológico a través del descubrimiento de sus funciones, su personalización y ajuste para lograr sus objetivos. La metodología utilizada fue comparativa y se centró en la búsqueda del aprendizaje en matemáticas, en un sentido tradicional y con la introducción del *software* GeoGebra, lo que evidenció relaciones que logren establecer en el dominio cognitivo. Los hallazgos de esta investigación cualitativa revelaron diferencias cognitivas en el uso del *software* GeoGebra, por lo que se propone el empleo óptimo según la dificultad que los estudiantes percibieron en los diversos temas matemáticos. Este grado de dificultad percibido

por los estudiantes permitió una clasificación en cinco perfiles asignados a los niveles de instrumentalización e instrumentación logrados en los primeros ejercicios. La aplicación de esta clasificación permitió un uso terso y eficiente del *software* GeoGebra por parte de los estudiantes y generar en ellos mayor seguridad y la posibilidad de terminar los procedimientos de cada ejercicio. Como resultado de la propuesta, se logró que el grupo de estudiantes que anteriormente no lograba terminar los ejercicios o que los hacían de manera incorrecta terminaran el tercer ejercicio. La construcción de un instrumento GeoGebra-estudiante, con su uso adecuado, nos permite equilibrar el desempeño de los estudiantes en un grupo no homogéneo, en el aspecto cognitivo. Los resultados de la presente investigación son importantes porque permiten fortalecer el uso y exploración de la tecnología en el aula de matemáticas para evaluar sus bondades y obstáculos en el proceso de enseñanza-aprendizaje.

**Palabras clave:** parábola, GeoGebra, instrumentalización, instrumentación, cognición.

## Abstract

This research analyzes the cognitive consequences that generate the GeoGebra *software* use as a technological instrument in the learning of mathematics, specifically in the teaching of Analytic Geometry in high school. This research uses, like theoretical framework, the theory of the Instrumentation because it is pertinent in the process of identification of the limitations and possibilities of a technological instrument by means of the discovery of its functions, its personalization and adjustment to achieve the cognitive objectives. The methodology was comparative focused in the search of the learning in mathematics, in a traditional sense and with the introduction of the GeoGebra software, evidencing relationships that are able to establish in the cognitive domain. At level of discoveries cognitive differences are observed in this qualitative research with the use of the software GeoGebra and the traditional mathematics class, and it is intended the effective use of GeoGebra according to the difficulty that the students perceived in the different mathematical topics. This degree of difficulty perceived by the students allowed a classification in five profiles assigned to the instrumentalization and instrumentation levels achieved in the first exercises. The application of this classification allowed a smooth and efficient use of the Geo- Gebra software by the students and to generate in them greater security and the possibility of finishing the procedures of each exercise. As a result of the proposal, it was achieved that the group of students who previously was not able to finish the exercises or some done it in an incorrect way, they finished the third exercise. The construction of an instrument GeoGebra-student, with its appropriate use, allows us to balance the acting of the students in a non-homogeneous group, in the cognitive aspect. The results of the present investigation are important because

they allow us to strengthen the use and exploration of the technology in the classroom of mathematics, to evaluate their kindness and/or obstacles in the teaching-learning process.

**Keywords:** Parable, GeoGebra, instrumentalization, instrumentation, cognition

## Resumo

A presente investigação analisa as consequências cognitivas geradas pela utilização do software GeoGebra como instrumento tecnológico para a aprendizagem da matemática, especificamente na unidade de aprendizagem de Geometria Analítica do nível secundário superior (ensino médio). Para isso, utilizou-se a teoria da instrumentação como referencial teórico por ser relevante reconhecer as limitações e possibilidades de um instrumento tecnológico através da descoberta de suas funções, sua customização e ajuste para atingir seus objetivos. A metodologia utilizada foi comparativa e focada na busca da aprendizagem em matemática, no sentido tradicional e com a introdução do software GeoGebra, que mostrou relações que se estabeleceram no domínio cognitivo. Os achados desta pesquisa qualitativa revelaram diferenças cognitivas na utilização do software GeoGebra, portanto o uso ideal é proposto de acordo com a dificuldade que os alunos perceberam nos diversos tópicos matemáticos. Este grau de dificuldade percebido pelos alunos permitiu uma classificação em cinco perfis atribuídos aos níveis de instrumentalização e instrumentação alcançados nos primeiros exercícios. A aplicação desta classificação permitiu uma utilização tranquila e eficiente do software Geo-Gebra pelos alunos e gerou maior segurança nos mesmos e possibilidade de realização dos procedimentos de cada exercício. Como resultado da proposta, o grupo de alunos que anteriormente não conseguiu finalizar os exercícios ou que os fez de forma incorreta conseguiu finalizar o terceiro exercício. A construção de um instrumento GeoGebra-aluno, com a sua utilização adequada, permite-nos equilibrar o desempenho dos alunos num grupo não homogêneo, no aspecto cognitivo. Os resultados desta pesquisa são importantes porque permitem fortalecer o uso e a exploração da tecnologia na sala de aula de matemática para avaliar seus benefícios e obstáculos no processo de ensino-aprendizagem.

**Palavras-chave:** parábola, GeoGebra, instrumentalização, instrumentação, cognição.

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## Introduction

Higher secondary education (NMS) plays an essential role in preparing students who aspire to enter higher education (NS) and for their future in the world of work. Therefore, it is crucial to stay updated in the use and exploration of technology, specifically in topics related to the teaching of mathematics, where there are numerous alternatives in the form of assistants, such as Mathlab , Mathematica , Cabri Geometry , Maple, among others. These programs are effective, but often require paid licenses, which presents challenges for their implementation in public education systems. In fact, if licenses are purchased, their use is often restricted to computer lab environments, which does not match the typical environment of a mathematics class.

In this panorama, GeoGebra emerges as a solution that combines open source *software* and mathematics for free. Indeed, GeoGebra offers a dynamic platform that covers various branches of mathematics, including algebra, geometry, trigonometry, analytical geometry, differential calculus, integral and statistics. Furthermore, it allows the efficient visualization and manipulation of abstract geometric objects ( Alkhateeb and Al- Duwairi , 2019; Emaikwu *et al .* , 2015; Soetadiana , 2014), thus helping to improve students' mathematical skills, including mathematical reasoning and problem solving ( Granberg and Olsson , 2015; Tamam and Dasari , 2020), facilitates the learning process by offering enriched content ( Yorganci , 2018), as well as the manipulation and exploration of representations ( Takaci *et al .* , 2015) and the formulation of conjectures and arguments ( Baccaglini -Frank, 2019).

In particular, GeoGebra stands out in addressing topics such as straight lines, circles, parabolas, hyperbolas and ellipses, including their canonical and general representations. It is also very useful for teaching concepts related to differential calculus and the geometric interpretation of the derivative, as well as the identification of maximums and minimums of functions, among other aspects (Borbón, 2010; Ibarra et al., 2018 ) . Furthermore, it has been shown to have a positive emotional impact on the learning process, as noted by Tamam and Dasari (2021).

The basic use of the *software* focuses mainly on Euclidean geometry and analytical geometry, as detailed in the research of Carrillo and Llamas (2009). Their study describes the program's interface and how to use its various toolbar icons, as well as the advanced graphing features offered by the computer version of the *software* . Other reports, such as that of Borbón (2010), focus on continuous algebraic functions and their dynamic representation in graphs.

In the field of teaching and didactic strategies related to the use of GeoGebra in its computer version, there are studies such as that of Villalobos *et al .* (2017) and the report by

Iranz and Fortuny (2009), which describe teaching strategies and how the use of GeoGebra improves student learning in subjects such as algebra, analytical geometry, and differential calculus. Additionally, the use of this tool has been investigated in teaching specific topics, such as volumes of revolution in integral calculus ( Machromah *et al* ., 2008).

GeoGebra has also been the subject of study regarding its impact on mathematical visualization and teachers' beliefs about its usefulness in teaching mathematics. For example, Dockendorff and Solar (2018) explored how *software* influences mathematical visualization and teachers' perceptions of its benefits. Likewise, Villagrán *et al* . (2018) investigated its use in applications of vector calculus, differential and integral calculus with one and more variables using a semiotic system. Finally, the book by Estrada *et al* . (2017) focuses exclusively on the issue of limits using GeoGebra.

Currently, educational mobile applications are considered tools that improve mathematics performance in the classroom. The positive impact of these for the acquisition of mathematical skills has been supported by studies such as that of Castillo and Arteaga (2021). In this sense, GeoGebra offers a version for mobile devices that can be used in the classroom, even without an internet connection, which makes it attractive for exploring mathematical concepts with smartphones (Fioriti, 2017; Ibarra, 2019) .

To take advantage of GeoGebra in the classroom, workshops have been promoted to train teachers in its use on mobile devices, which allows activities to be introduced into the mathematical learning environment and convert the classroom into a space for experimentation (García *et al*., 2018; Ibarra, 2019). This not only improves understanding of concepts but also empowers students by allowing them to direct their own learning and monitor their performance (Mohamed and Mohammed, 2013).

Having explained the above, it can be indicated that this research focuses on the use of GeoGebra in the Analytical Geometry class and its impact on the cognitive development of students. Specifically, the study focuses on the construction and interpretation of graphical representations of algebraic operations, obtaining equations from graphs and the conversion of canonical equations to general expressions. Detecting errors and improving problem-solving procedures is also discussed. The objective is to identify the levels of instrumentalization and instrumentation that GeoGebra allows and that are not achieved in a traditional way, for which the instrumental theory of Rabardel (1999) has been taken as a reference. In this sense, the question asked is the following: what are the levels identified by the instrumentalization and instrumentation with the use of GeoGebra that are not always achieved in a traditional way?



## Theoretical elements

To analyze the cognitive processes resulting from the use of GeoGebra in the Analytical Geometry learning unit and demonstrate the mathematical competencies acquired, corresponding to the NMS, a theoretical framework is required that allows managing concepts extracted from the use of technology and adapting them to didactic objectives. regarding the school environment that arise when implementing the aforementioned *software*. The relationship established between the technological device and the user for learning purposes is not superficial, if it is based on the idea that modifying the environment with the use of the software in which the students are placed develops in them other *competencies* and problem-solving heuristics, different from traditional ones, whose detection requires different concepts that allow the student- *software* binomial to be adequately understood. In this sense, the concept selected for this relationship is mediation.

Vygotsky (2018) introduced the concept of psychological instrumental mediation at the beginning of the 20th century through the use of language, its signs and conceptual relationships. This mediation, in analogy with the function of physical instruments in modifying the natural environment, is used to achieve higher knowledge.

The inclusion of the instrument in the behavioral process first causes the activity of a whole series of new functions, related to the use of the aforementioned instrument and its handling. Secondly, it suppresses and makes unnecessary a whole series of natural processes, whose work begins to be carried out by the instrument. Thirdly, it also modifies the course and the different characteristics of all the psychic processes that are part of the instrumental act, replacing some functions with others (Vygotsky, 2018, p. 67).

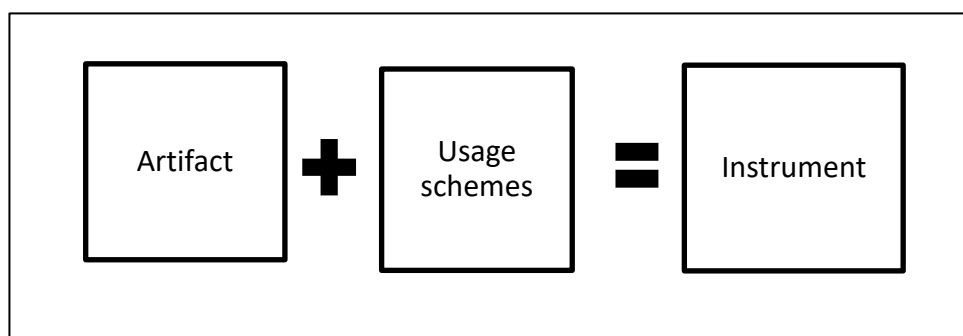
Rabardel (1999b) has expanded Vygotsky's ideas on mediation with his instrumental theory, which is not limited to a specific type of instrument, whether technical or psychological, in its mediating function, but encompasses all types, independently. of their nature, symbolism or conceptual character, whether internal or external, and whether directed at individual or collective behavior.

In the case of mathematics teaching, the use of GeoGebra as a mediation tool in the learning process can be understood through the theoretical framework of Rabardel 's instrumental theory (1999a). This uses clearly defined concepts, such as artifact, instrument, instrumental genesis, among others, to structurally support the objectives of this article in the cognitive field, especially in relation to the interaction between a technological artifact and the user.

A prominent point in this theory is the concept of instrument, which is considered a mixed entity composed of both the part of the artifact and the associated utilization schemes, which are the result of the construction of the subject to achieve a specific objective. This means that the same artifact can have multiple different utilization schemes, leading to different ways of achieving the desired goal, in this case, mathematics proficiency.

So how can an instrument act as a mediator between the subject and the object? The key lies in the objective, which prompts the subject to use the artifact in a particular way, based on a specific schema that has been learned, as illustrated in Figure 1.

**Figure 1.** Instrument components



Source: Own elaboration based on Rabardel (1995)

The process by which an artifact becomes an instrument was called *instrumental genesis* by Rabardel (1995) , and consists of the interaction of instrumentalization and instrumentation, which are key concepts of instrumental theory and that require to be defined in the following way. :

**Instrumentalization:** It is the process of identifying the limitations and possibilities of an artifact through the discovery of its functions, its customization and adjustment to achieve its objectives, that is, it is the discovery of the properties of the artifact by the subject (of the subject to the artifact). This process makes a clear distinction between the novice and the expert ( Sua and Camargo, 2019).

**Instrumentation:** The progressivity in the knowledge of the artifact (commands and functions) generates rearrangement of the subject's use schemes (from the artifact to the subject), which in turn makes the instrument more complex (Rabardel, 1999a) and generates paths to resolution of tasks (heuristics).

## Levels of competence by instrumentalization and instrumentation

Instrumentation and instrumentalization make up a bidirectional process, but they generate indicators of different levels according to their own use schemes, which allow the desired objectives to be achieved or not (table 1). In this sense, the adaptation of the competencies with the instrumental achievements is not total, so the support of what was achieved in the notebook is valid and accompanies the process carried out in GeoGebra .

**Table 1.** Levels of competence by instrumentalization and instrumentation

Process level	Using GeoGebra	Competencies of the NMS, IPN study program
Instrumentalization 1	GeoGebra commands and the corresponding geometric concept.	
Instrumentalization 2	Identification of actions, such as fixing points, dragging points, mirroring points with respect to an axis, rotation, translation, obtaining angles, etc.	Analytic geometry Particular competence 1. Solve problems of geometric places, particularly straight lines, using the properties of the Cartesian plane in academic and social situations.
Instrumentalization 3	Carry out instructions in GeoGebra such as graphing points, joining points, obtaining slopes, areas, perimeters, etc.	
Instrumentation 1	Interpret the equation and/or graph of a line based on the usual data of an academic or applied problem in Analytical Geometry.	
Instrumentation 2	Obtain and interpret the equation and/or graph of a conic from the usual data of an academic or applied problem of Analytical Geometry.	Analytic geometry Particular competence 2. Solve problems involving quadratic equations and their graphic representation, by identifying the specific elements of each of the conics, in academic and social situations.



Instrumentation 3	Handle two or more related equations and obtain their graphic representation from the usual data in an applied parabola problem in Analytical Geometry.	
Instrumentation 4	Use the dynamic geometry of Geogebra to interpret parabolic movement based on the variation of a given parameter in Analytical Geometry.	

Source: Own elaboration with information from Coca (2022, p. 21)

Once the basic elements of Rabardel's (1995) instrumental theory have been reviewed, we proceed to identify the relevant conceptual categories to examine the cognitive processes developed by students during instrumentation from a proposal for using GeoGebra.

## Methodology

The methodology used in this study had a comparative nature (Rosas and Pardo, 2013), which “presupposes an a priori comparison of inequality or difference” (Raventós, 1983, p. 63). In this research, the objective of promoting learning in mathematics is shared, both in a traditional approach and with the introduction of GeoGebra, for which the variety of relationships that can be established in the cognitive context is highlighted. Likewise, it is important to highlight that the conclusions drawn are not intended for generalization, but rather focus on the detailed study of a specific learning situation in mathematics in a particular environment, as reflected in the intervention and the corresponding analysis.

Now, the research on the impact of the GeoGebra *software* in Analytical Geometry classes at the upper secondary level (NMS) of the National Polytechnic Institute (IPN) required considering the specific environment in which the present study was carried out. That is, the real conditions that were experienced during the pandemic, which forced classes to be developed in a virtual environment and then in a hybrid modality.

This situation, far from detracting from value, increases the interest of the research in terms of the actions and results of the participants. In addition, the rigidity of the study programs and their conformity with institutional guidelines must be considered, which determines that GeoGebra does not have a main or specific role in the class, but rather depends on the use and time that students assign to it.

## Design of the investigation

The research was carried out with the participation of a group of 50 third-semester students of the upper secondary level (NMS) of the IPN, who voluntarily agreed to be part of this study after an informative talk about their characteristics. Although the teacher-researcher had a total of 120 potential candidates, data was only collected from these 50 participating students. The students themselves completed the registration, assigning them a number that related the survey to problem solving. Specifically, the version of the GeoGebra *software* designed for mobile devices was used, which allowed its use in class without requiring Internet access after previously loading it at home.

The data was obtained from two activities related to the topic of the parable, which were carried out using both the workbook and the GeoGebra *software*, according to the preference of each student, either before, during or after completing work in the notebook. Then, a third activity was applied based on the cognitive profiles that emerged from problem solving with and without the use of the *software*. The objective was to propose the right moment to use the dynamic *software* in order to optimize the results.

The ages of the participants ranged between 15 and 16 years, and they were enrolled in the Analytical Geometry learning unit in the third semester of the Center for Scientific and Technological Studies number 14 (CECyT 14) of the NMS of the IPN, in Mexico. It is important to highlight the dual role of the teacher-researcher in the creation of this educational reality by introducing technology through the GeoGebra *software* in a classroom environment, following a qualitative approach in the research. The study was divided into three phases.

### First phase: framing in the mathematics classroom

The first phase focused on training the student to use and explore the *software* in mathematics class through simple sequences and exercises. The students were in constant communication during two weeks of training to endorse their opinions on the matter and support each other. This first phase was training only and the results were not recorded.

### Second phase: development of two activities

Two problems related to the theme of the parable were raised. The first activity consisted of identifying the elements of a parabola with a symmetry axis parallel to “x”, and with a vertex outside the origin and its corresponding graphic representation. The activity was carried out in the workbook using millimeter sheets to graph, as well as the *software* GeoGebra. The skills acquired in each procedure during the experience were recognized.

The second activity focused on a contextualized situation in which the concept of the parabola was applied to the subject of physics, specifically to analyze parabolic motion. Solving this activity required prior knowledge in physics, as well as understanding of the basic concepts related to parabolas. Furthermore, this activity had significant scope, as the use of GeoGebra allowed the creation of dynamic representations of movement.

The role of the teacher-researcher was fundamental in this second phase, since at the end of the sessions, the students submitted their work and products obtained for evaluation. In addition, they provided electronic captures of the generated graphical representations and a record of the commands used in GeoGebra. They also indicated whether they had explored the *software* before, during, or after completing the workbook activity. This aimed to develop a typology that described how students used GeoGebra, considering the digital skills they had acquired and the procedures they followed during the solving process.

### **Third phase: conclusion**

At the end of the proposal to incorporate GeoGebra, considering the various characteristics of the students, a third specific activity was designed for the group. This focused especially on those students who had not completed one or both of the first two activities, or whose submissions, although on time, contained incorrect solutions. These deficiencies could be attributed to several factors, such as limited time, basic skills in using GeoGebra, basic mathematical knowledge about the topics covered, or even attitudes towards learning mathematics that could have been influenced by the use of the *software*. In the third activity, the students used the *software* according to the classification they had achieved in phase 2, that is, according to the level of cognitive development achieved.

### **Results and analysis**

With the purpose of using the *software* as a methodological mediation instrument for teaching mathematics, three activities related to the topic of the parabola were presented in the NMS Analytical Geometry course. The first activity had to do with the construction of a parabola with its vertex located outside the origin of coordinates and its symmetry axis parallel to the "x" axis. The second activity focused on the construction of a parabola with the vertex outside the origin and its axis of symmetry parallel to the "y" axis, in the context of parabolic motion in physics. The last activity addressed the location of the elements of a circle. These activities are described below.

First activity: *Determine the equations of the directrix and parabola whose vertex is the point  $(-2,4)$  and whose focus is located at  $(-5,4)$ . Obtain four points added and make the graph.*

The particular competence acquired by students focuses on the construction and interpretation of a horizontal parabola and its graphic representation. This involves working with quadratic equations and understanding the particular elements of conics in academic and social situations, at a basic level, as established in the study program.

The use of GeoGebra—whether before, during or after the analytical and graphical resolution process in the workbook—is at the discretion of the student, as it depends on his or her prior knowledge and familiarity with the essential commands and capabilities offered by the *software*. As the student becomes familiar with it, it becomes a powerful tool that adapts to their needs (Rabardel, 1995). Thus, the student begins the exploration of the various commands, which contributes to making heuristic decisions and constructing the required graphic representation, highlighting the use of the following elements: axis of symmetry of the parabola to reflect through axial symmetry the various symmetrical ordered pairs, exploration of the intersection to recognize the straight side, points of the trace of an axis perpendicular to the axis of symmetry and the focus of the parabola to be discussed in plenary.

On the other hand, although the graphs generated by the students with the *software* are visually similar, the procedures and knowledge used vary significantly, since some students used strategies based on symmetry to reflect points, while others preferred to create the graph point by point sequentially. Likewise, there were those who generated the graph from the equation obtained in their notebook, while others used specific GeoGebra commands to graph and then locate the points and their intersections with the axes.

The results indicate that students acquire three main competencies when working with GeoGebra. First, they develop an instrumentalization competency by relating *software commands* to relevant analytical and geometric concepts. Secondly, they acquire instrumentalization skills associated with the manipulation of points, their translation and reflection on different axes of symmetry, evaluating the results of these actions. Finally, they develop instrumentation skills by interpreting data and deducing the equation necessary to construct a graphical representation, in accordance with the basic approach of the Analytical Geometry program, which does not establish relationships with the different geometric loci.

## Comparison between work, development in the workbook and use in GeoGebra

Although the solution proposed in the notebook and its representation on the millimeter sheet resemble the graph generated in GeoGebra, there are significant differences in the approaches due to several circumstances. One of them is the reluctance of students to make changes to millimeter sheets due to concerns about making mistakes and having to start over. Instead, the *software* allows you to make adjustments seamlessly and explore the symmetry of the figure without the fear of having to redo the entire process. These factors suggest that mediating learning through GeoGebra does not necessarily develop the same skills, at least in a first approach.

Second activity: In a context that involves both uniform rectilinear motion (MRU) and uniformly accelerated rectilinear motion (MRUA), the student explores the formulas and physical quantities related to these phenomena separately. In this process, the student relies on his previous knowledge about parabolic motion, which is understood as the combination of two types of motion: a uniform rectilinear motion (MRU) with the velocity component constant in the horizontal component, and a uniformly accelerated rectilinear motion (MRUA) subject to the action of gravity in the vertical component. At this point, the teacher-researcher emphasizes the function and importance of the parameter “*t*”, which is represented in both the MRU and the MRUA, as well as to obtain a parabola.

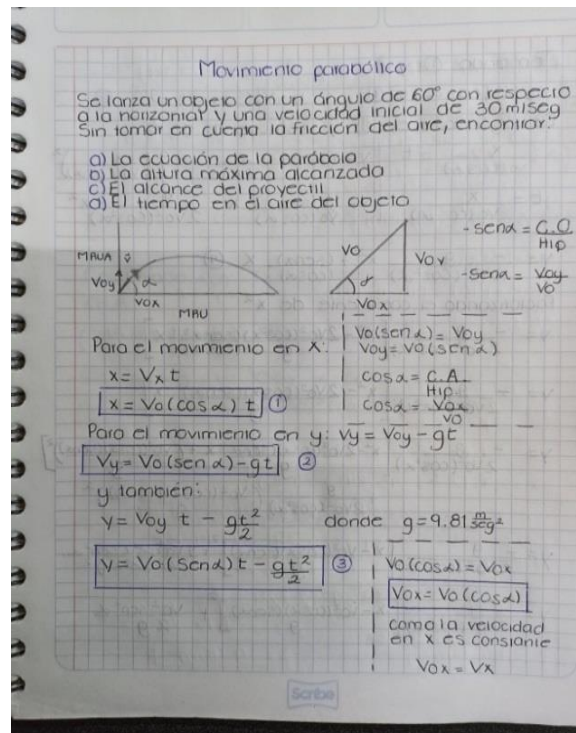
As part of the conclusion of this stage, a problematic situation is posed with the purpose of establishing connections between the concepts and procedures of Analytical Geometry, specifically in relation to parabolas with vertical axes of symmetry, and the principles related to parabolic motion. This approach represents an application of Analytical Geometry in the context of physics. The text that describes this problem is the following:

*An object is thrown at an angle of  $60^\circ$  with respect to the horizontal and an initial speed of 30 m/s, without taking air friction into account. Find a) the equation of the parabola, b) the maximum height reached by the object, c) the range of the projectile and d) the time in the air of the object (Coca, 2022, p. 21).*

The student develops the analysis of the situation in their notebooks (figures 2 and 3), using vector decomposition, and performs the necessary algebraic steps to establish the equation that involves the variables “*x*”, and represents a parabola.

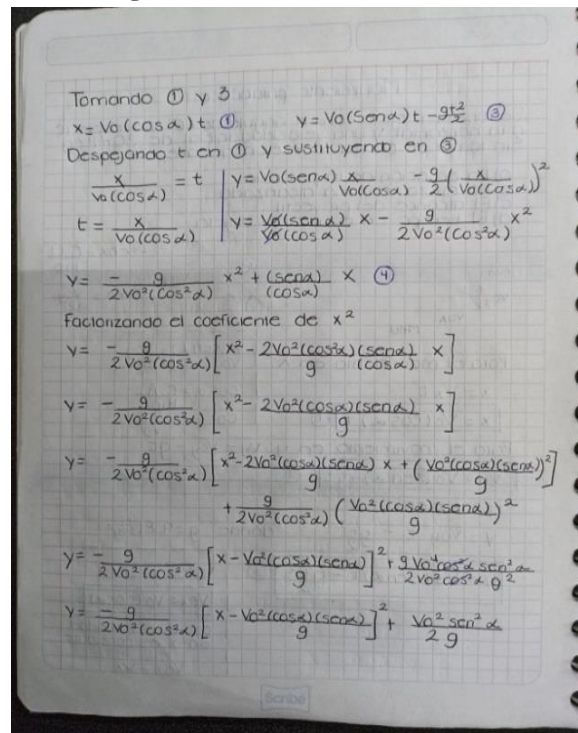


Figure 2. Parabolic movement



Source: Analytical procedures of students to address parabolic movement

Figure 3. Parabolic movement



Source: Continuation of the analytical procedure of the student body to address the parabolic movement



Subsequently, explore GeoGebra with the objective of graphing the parabola and proceed to build the parametric equations (as a function of time) for the “x” coordinate, as well as for the “y” coordinate (equations 2 and 3). The procedure is presented in Figure 4.

**Figure 4.** Construction of parametric equations

$$x = v_0 (\cos \alpha) t \quad (1)$$

where, substituting the value of the initial velocity and angle yields the expression

$$x = 30 (\cos 60^\circ) t$$

$$x = 30 (0.5)t$$

$$x = 15 t \quad (2)$$

for "y"

$$y = v_0 (\sin \alpha) t - gt^2/2 \quad (3)$$

where, substituting the value the initial velocity, the angle and the value of "g" is obtained:

$$y = 30 (\sin 60^\circ) t - (9.81) t^2/2$$

$$y = 25.98 t - 4.90 t^2 \quad (4)$$

from 2 and 4 the point is obtained depending on the parameter:

$$(15t, 25.98 t - 4.90 t^2)$$

This point can be inserted with Geogebra in the part corresponding to algebra, where the parameter can be valued, knowing that the time in the air of the object is 5.29 sec remaining from.

$$t=0 \text{ hasta } t=5.29\text{seg.}$$

Previously we graph the maximum point corresponding to the vertex of the parabola which is:

$$(39.7, 43.4)$$

as well as, the point corresponding to the maximum range, which is the plane:

$$(79.4, 0)$$

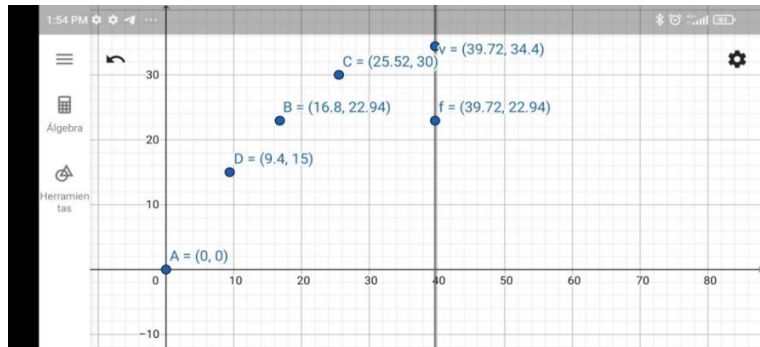
Note: Exposition of a student's procedure in the construction of parametric equations

Most students presented analytical procedures. It is worth mentioning that the time spent in its execution was more than an hour and a half, although other students requested support from their classmates to complete the process due to limitations in the algebraic handling of mathematical expressions. Generally, and even in these cases, the learning was enthusiastic but overwhelming.

Regarding the use of GeoGebra, two types of procedures developed by the students and, therefore, two different types of instruments were presented. Some of them limited themselves to what they had learned in class, and essentially reproduced the same procedure

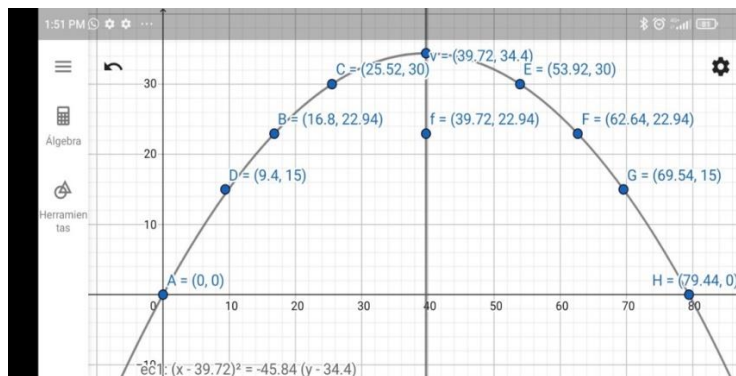
outlined in the notebook with the acquired commands, including the symmetrical reflex. The dynamic *software* was a support to corroborate the points to be graphed in the notebook (previously) or to compare graphs at the end (posteriorly) to reach a level of instrumentation 2 and instrumentalization 2 (figures 5 and 6).

**Figure 5.** Points of parabolic motion with GeoGebra



Note: Presentation of the use of the software to obtain the graph of the parabola from the usual data of the situation by the student

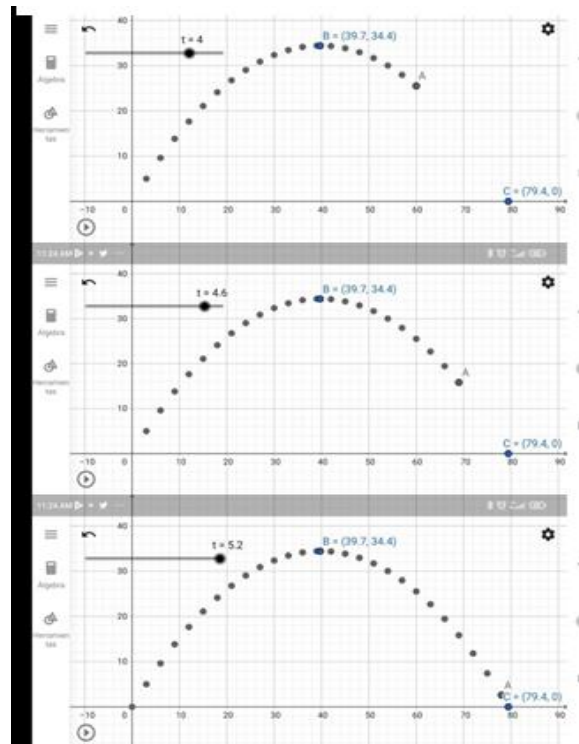
**Figure 6.** Parabolic movement with GeoGebra



Note: Exposure of the use of the software to identify various points, drag points and reflection of points with respect to an axis, by the student

A small number of students analyzed the different dynamic particularities of the program and achieved the parametric representation (figure 6). This procedure generated the equations of motion for both “x” and “y” based on the parameter “t”, which can be entered as a point  $(x(t), y(t))$  in the algebraic section of GeoGebra and given values to the parameter “t” from 0 (there are no negative times) to 5.29, where the intersection with the “x” axis occurs, allowing the dynamic application to run that shows the development of the parabolic movement (figure 7).

**Figure 7.** Dynamic representation of movement with GeoGebra



Note: Dynamic particularities of the software to obtain the parametric representation by the student

By developing the exercise in his notebook and graphing it on a millimeter sheet, the student involves a parabola with a symmetry axis parallel to the "y" axis, as well as its graphic representation and identification of its specific elements to apply them to a physics problem such as movement parabolic. The exercise corresponds to the particular competence: the student must be able to obtain and interpret the equation and/or graph of a conic from the usual data of an Analytical Geometry problem and its relationship with the different areas of knowledge (see Table 1, theoretical references).

Based on experience, it is considered a broad learning, and at a disciplinary level it corresponds to the learning units of Algebra (right triangle, graphs, equations of first and second degree) and Analytical Geometry (from the general equation to the canonical equation, graphing and interpretation of variables), in a constant change between the learning unit of Analytical Geometry and the physics subject. Regarding the physics learning unit, the topics MRU, MRUA, vectors and their combination in parabolic motion are addressed.

With the recognition of the basic commands of the *software* and using the artifact at this stage as an instrument (commands + user skills), the students undertook their exploration in greater depth until reaching instrumentation level 3, which was reached, after several setbacks, in the dynamic representation of parabolic movement. The evaluation of the exercises by the teacher-researcher allowed the cognitive classification to continue, the results of which are presented in Table 2.

**Table 2.** Cognitive results

Instrument Levels	<i>Elementary</i>	<i>Essential</i>	<i>Half</i>		<i>High</i>
	GeoGebra instructions for plotting points and plotting graphs from tabs	Explore the <i>software</i> as necessary to use the features of the functions and relate them	Explore the <i>software</i> in depth including axial symmetry and equation-graph correspondence		Explore the <i>software</i> completely, including dynamic features
Cognitive outcome	Instrumentalization: level 1	Instrumentalization : levels 1 and 2 Instrumentation level 1	Instrumentalization: levels 1,2 and 3 Instrumentation : level 1 and 2	Instrumentalization: levels 1,2 and 3 Instrumentation : level 1 2 and 3	Instrumentalization: levels 1,2 and 3 Instrumentation : level 1, 2, 3 and 4
Process	Incomplete incorrect exercises	Partial exercises Complete	Complete exercises	Complete exercises	Complete exercises. GeoGebra dynamics
Time of use of GeoGebra decided by the student himself	Before, during or after carrying out the exercise in the notebook	Before or during the exercise in the notebook	Before or during the exercise in the notebook	After completing the exercise in the notebook	Before, during or after carrying out the exercise in the notebook

Source: Own elaboration with information from Coca (2022, p. 58)

In a certain sense, the graphic expressions exposed in the notebook and in GeoGebra, although they manifest similar movements, are not cognitively equivalent. The fixed representation of the parabolic motion in the GeoGebra representation only serves verification or prediction functions, compared to the fixed representation in the notebook. However, the change from the fixed representation of the notebook is presented in contrast to the dynamic representation provided by the GeoGebra graphical interface.

By exploring and discussing the different representations of the parabola through the use of the instrument, and after various strokes, the students recognized the dynamic representation of parabolic motion. However, this caused frustration for some participants due

to the obvious contrast between the intense work required to solve in the notebook, and the parametric representation of GeoGebra.

Learning the spatio-temporal evolution of parabolic motion is reinforced with the use of GeoGebra, which constitutes a scenario that goes beyond traditional teaching. However, the mathematical learning of graphic study does not replace, but rather complements, the treatment of the transformation of the general parabolic equation into its canonical expression to identify vertex and focus, which the student develops in the notebook.

According to the levels of use of the instrument (elementary, basic, medium and high) that are generated with the cognitive result (level of instrumentalization and instrumentation) and the conclusion or not of the complete exercise, the five cognitive profiles are proposed (table 3), as well as the moment that is considered appropriate for the use of GeoGebra in carrying out the third activity and the objective of the proposal.

**Table 3.** Proposal for cognitive profiles

Levels	<i>Elementary</i>	<i>Essential</i>	<i>Half</i>		<i>High</i>
	GeoGebra instructions for plotting points and plotting graphs from tabs	Explore the <i>software</i> as necessary to use the features of the functions and relate them.	Explore the <i>software</i> in depth including axial symmetry and equation-graph correspondence.		Explore the <i>software</i> completely, including dynamic features.
Cognitive outcome	Instrumentalization : level 1	Instrumentalization : levels 1 and 2 Instrumentation level 1	Instrumentalization : levels 1,2 and 3 Instrumentation : level 1 and 2	Instrumentalization : levels 1,2 and 3 Instrumentation : level 1 2 and 3	Instrumentalization : levels 1,2 and 3 Instrumentation : level 1, 2, 3 and 4
Student	Profile A	Profile B	Profile C	Profile D	Profile E
Proposal for the moment of use of the use of GeoGebra	After completing the exercise in the notebook with additional time (30 minutes).	Before carrying out the exercise in the notebook with additional time (20 minutes).	During the exercise in the notebook .	After completing the exercise in the notebook.	During the exercise in the notebook.
Goal of the proposal	Build an elementary instrument, do	Build a basic instrument to rely in advance	Performing a simultaneo	Perform both activities	Build a highly efficient instrument that

	not be affected in completing your mathematical tasks.	on the graphical representation of GeoGebra without being affected by the time usually designated to carry out the exercise.	us development in the notebook and GeoGebra, he was able to build an instrument and was supported to obtain satisfactory results.	separately, use GeoGebra to check your results in the notebook.	can carry out simultaneous development and obtain a graphical solution that is mathematically more complete than the one requested.
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Source: Own elaboration

Third activity. The activity poses the challenge of determining the center and radius necessary to construct the graphic representation of a circle. This is achieved using the line that passes through the midpoint of each side of the triangle and is perpendicular to said side. The center of the circle is located at the intersection of two of the algebraic expressions that represent the bisector lines. To do this, students have prior knowledge that includes understanding the equation of a line, as well as the canonical and general equations of a circle. However, the activity presents additional conceptual challenges by involving concepts such as midpoints, perpendicular lines, and intersection of lines. The text is the following:

*Determine radius, center and graph the circumference that passes through the points A( -2,-3), B(5,-2) and C(7,-6) using a triangle and bisectors to establish the circumcenter.*

Below is a student's procedure in his workbook (figures 8, 9, 10 and 11).



**Figura 8. Activity 3, follow-up 1**

Encontrar el centro, radio y gráfica de la circunferencia que pasa por los puntos: A(-2,-3), B(5,-2) y C(7,-6) usando un triángulo, sus mediatrices y obteniendo el circuncentro.

Localizamos los puntos medios y las pendientes:

(AB)

$$\begin{aligned} &(-2, -3) \quad (5, -2) \\ &(x_1, y_1) \quad (x_2, y_2) \end{aligned}$$

$$x_{p.m.} = \frac{x_1 + x_2}{2} \quad y_{p.m.} = \frac{y_1 + y_2}{2}$$

$$x_{p.m.} = \frac{-2 + 5}{2} \quad y_{p.m.} = \frac{-3 + (-2)}{2}$$

$$x_{p.m.} = \frac{3}{2} \quad y_{p.m.} = \frac{-5}{2}$$

$$x_{p.m.} = 1.5 \quad y_{p.m.} = -2.5$$

$$(1.5, -2.5)$$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AB} = \frac{-2 - (-3)}{5 - (-2)}$$

$$m_{AB} = \frac{-2 + 3}{5 + 2}$$

$$m_{AB} = \frac{1}{7}$$

Primera mediatriz

$$(1.5, -2.5) \quad m = -7$$

$$(x_1, y_1)$$

Ecuación:

$$y - y_1 = m(x - x_1)$$

$$y - (-2.5) = -7(x - 1.5)$$

$$y + 2.5 = -7x + 10.5$$

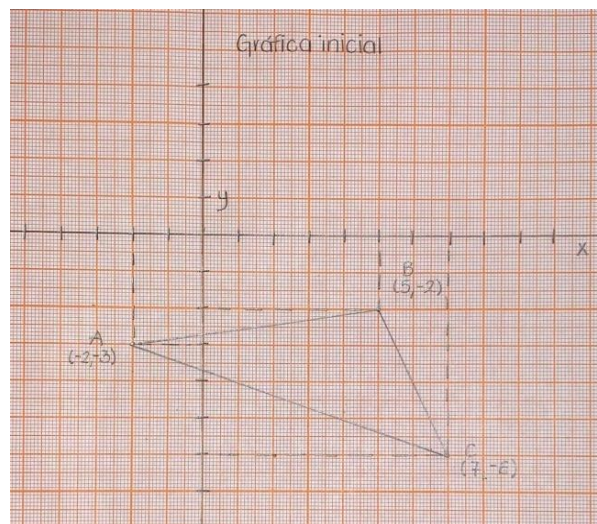
$$7x + y + 2.5 - 10.5 = 0$$

$$7x + y - 8 = 0 \quad (1)$$

Ecuación de la 1ª mediatriz

Note: Procedure of a student in the workbook, note 1

**Figure 9. Activity 3, follow-up 2**



Note: Procedure of a student in the workbook, note 2

**Figure 10.** Activity 3, follow-up 3

Tomamos 2 de las 3 ecuaciones, por ejemplo, ① y ②

$$\begin{aligned} x(-2) \begin{cases} 7x+y-8=0 \\ -x+2y+14=0 \end{cases} & \quad \begin{cases} 7(2)+y-8=0 \\ 14+y-8=0 \\ 6+y=0 \\ y=-6 \end{cases} \\ \textcircled{1} \begin{cases} -14x-2y+16=0 \\ -x+2y+14=0 \end{cases} & \quad \begin{cases} -15x+30=0 \\ -15x=-30 \\ x=-30/-15 \\ x=2 \end{cases} \end{aligned}$$

El punto  $(2, -6)$  es el circuncentro

Para obtener el radio, sacamos la distancia entre el circuncentro  $(2, -6)$  y alguno de los puntos iniciales, por ejemplo  $(2, -3)$

$$\begin{aligned} (2, -6) & \quad (-2, -3) \\ (x_1, y_1) & \quad (x_2, y_2) \end{aligned} \quad \begin{aligned} r &= 5 \\ r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ r &= \sqrt{(-2 - 2)^2 + (-3 - (-6))^2} \\ r &= \sqrt{(-4)^2 + (-3 + 6)^2} \\ r &= \sqrt{(-4)^2 + (3)^2} \\ r &= \sqrt{16 + 9} \\ r &= \sqrt{25} \end{aligned}$$

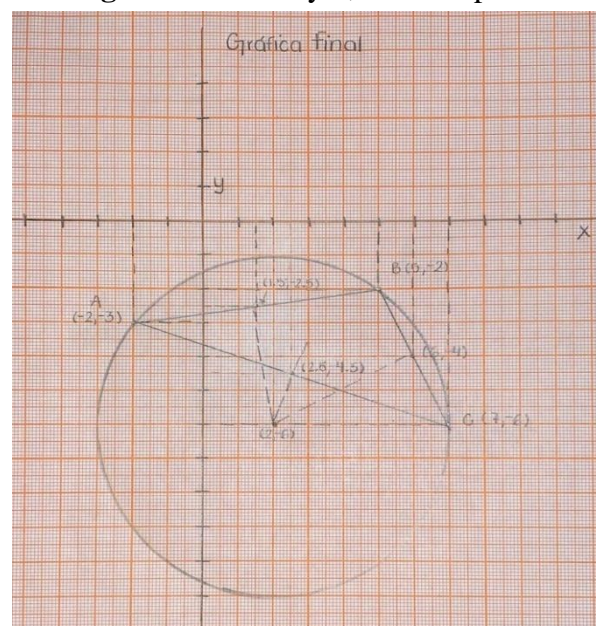
Centro  $(h, k)$   $r = 5$   
 $(2, -6)$

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 \\ (x-2)^2 + (y-(-6))^2 &= (5)^2 \\ (x-2)^2 + (y+6)^2 &= 25 \end{aligned}$$

Ecuación de la circunferencia

Note: Procedure of a student in the workbook, note 3

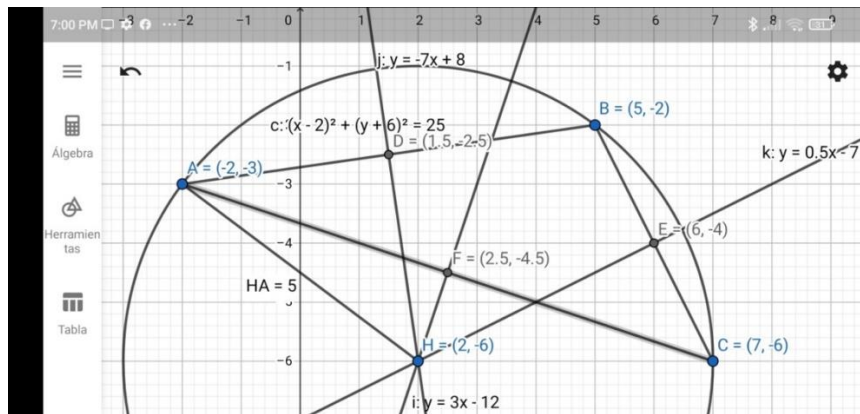
**Figure 11.** Activity 3, follow-up 4



Note: Procedure of a student in the workbook, note 4

Using GeoGebra, you started by plotting the three points to execute the *segment command* and draw the triangle; Later, he used the *middle or center command* to determine the midpoint of each side. Next, he used the *perpendicular command*, which allows us to draw the bisectors of the triangle and the *properties command* to draw its equations. Then, the point of intersection of the bisectors, the circumcenter, to obtain the circumference with the center and a point (figure 12).

**Figure 12.** Activity 3, GeoGebra



Note: Conclusion of activity 3 with the use of GeoGebra

Below are the cognitive profiles and the time that is considered appropriate for using the *software* GeoGebra in carrying out the third activity (table 4).

**Table 4.** Results of the proposed use of GeoGebra *software*

Student	Profile A	Profile B	Profile C	Profile D	Profile E
Process	Complete exercise, correct or incorrect	Complete exercise correct	Complete correct exercise	Complete correct exercise	Complete correct exercise
Cognitive outcome	Instrumentalization : level 1	Instrumentalization : levels 1 and 2 Instrumentation level 1 and 2	Instrumentalization : levels 1,2 and 3 Instrumentation: level 1 and 2	Instrumentalization : levels 1,2 and 3 Instrumentation : level 1 , 2 and 3	Instrumentalization : levels 1,2 and 3 Instrumentation : level 1, 2, 3

Source: Own elaboration

According to the classification presented in Table 4, the students who are in profiles C, D and E did not experience significant difficulties when carrying out the activity. Instead, profile E students explored the *software* while working in the notebook, applying the necessary mathematical operations. For their part, profile D students used GeoGebra after completing the activity in the notebook, mainly to verify the results obtained. As for profile C, they managed to combine the completion of their exercise in the notebook with the representation in GeoGebra, without facing notable obstacles.



However, profile A, which GeoGebra used after completing the activity in the notebook, failed to assign adequate meaning to the verification functions in GeoGebra and did not express a clear position on the differences between the results obtained in the notebook and in GeoGebra.

The results indicate cognitive changes in profile A students who managed to complete the activity; Profile B, although it made several errors, managed to correct them later. Profiles C, D and E worked without apparent difficulties. Students at risk of not achieving mathematical competencies are those belonging to profiles A and B, but their performance showed significant improvements during activity 3. Profile C appears to be in a stronger position, as students demonstrated mathematical performance acceptable.

In this context, it is important to note that the graphical representations presented in the workbook and on the mobile phone screen, although they represent similar movements, are not completely equivalent from a cognitive perspective. The static representation of parabolic motion in the notebook is similar to the static representation in GeoGebra and is primarily used for verification purposes. In contrast, the static representation in the notebook corresponds to the dynamic representation provided by the graph in GeoGebra. Students who managed to construct the graphical representation expressed satisfaction, while some students found the comparison between the work done in the notebook and the parametric representation in GeoGebra frustrating.

## Discussion

In mathematics class, students expressed various difficulties related to the procedures and the limited time to solve them. In addition, a lower skill in exploring the *software* was recognized. That is, a deficient instrument was built because, as Rabardel (1999) mentions, GeoGebra is an additional factor that hinders the time to complete the task, coupled with negative emotional stress in their mathematical activities.

In this sense, other students show incomplete processes when exploring the *software* as a complement, which prevents the immediacy of the graphic representation and makes their work in the notebook more laborious. The group of students who complete the activities manage to visualize the different geometric objects efficiently, which agrees with Emaikwu *et al.* (2015) and Alkhateeb *et al.* (2019). In general, these students accepted, explored and used GeoGebra in the tasks designed with the willingness to strive to investigate more functions and applications, thereby building a more complete instrument in the sense given by Rabardel (1999).

On the other hand, the students' collaborative work with GeoGebra promoted an environment of trust and a positive attitude towards the course (Er *et al.*, 2017; Yorganci , 2018). Likewise, while interacting with their mobile devices, students communicated with each other to clarify or refute the results obtained in their workbooks, even showing the screens of their devices. This enriched the discussions between classmates (Andrés et al., 2022), creating an optimal learning environment for the analysis and interpretation of the visual characteristics of algebraic expressions through the *software* . Additionally, this environment fostered deep and comprehensive learning of mathematical concepts ( Yorganci , 2018).

The use of GeoGebra therefore provided dynamic tools that made a significant difference in the understanding of parabolic motion (activity 2). By introducing the parametric coordinates of the parabola and allowing visualization of the sequence of motion, students were able to gain a dynamic understanding of the equations of motion for “x” and for “y” as a function of the parameter “t”. This contributed to strengthening their understanding of the dynamic structure and dynamic transitions in these representations, which in turn boosted the construction of their mathematical knowledge and helped them develop problem-solving skills (Granberg and Olsson , 2015 ; Öçal , 2017; Tamam and Dasari , 2020; Tatar, 2017).

Consequently, and considering the evidence of the procedures used in solving the proposed activities, it can be concluded that the application of GeoGebra is effective in improving the problem-solving capacity in Analytical Geometry courses, while at the same time promoting the development of mathematical skills in students ( Sudihartinih and Wahyudin , 2019).

In addition to the above, it is essential to document the results of this intervention and its implications, which will allow progress in the use of technology in the mathematics classroom. Therefore, the implementation of changes in the area's study programs could be suggested, giving teachers greater flexibility to integrate technology not as a complement, but as a substitute for traditional methods in order to evaluate its impact.

## Final considerations

Although this research focused on maintaining the use of pencil and notebook as an initial condition, with GeoGebra mainly fulfilling a function of cognitive support for rapid graphic visualization, a significant contribution is observed in the cognitive aspect, especially in the use of axial symmetry for point reflection.

In relation to the use of GeoGebra with its dynamic capabilities, the results coincide with the findings presented by Tamam and Dasari (2020), since the *software* makes an important difference in the understanding of parabolic motion by introducing the parametric coordinates of the parabola and allowing the visualization of the movement sequence. However, it is important to highlight that not all students handled the equations adequately, and only some achieved the dynamic representation.

On the other hand, a classification of students was proposed according to their focus on solving exercises and the appropriate moment was identified for the GeoGebra *software* to support them, which generated greater confidence in them and the possibility of completing the procedures of each exercise.

In summary, as a result of this proposal, the group of students who previously had difficulties completing the exercises (profile A) or made errors in their resolution (profile B) managed to complete the third exercise. Although some students in profile A still made errors, all students in profile B obtained correct results. On the other hand, students in profiles C, D and E maintained solid performance, as they had already demonstrated in exercises 1 and 2. In short, the creation of a GeoGebra-student instrument, when used appropriately, allows leveling the performance of students in a group characterized by its cognitive diversity.

## Future lines of research

Future research could focus on the integration of technology not as a complement, but as a replacement for traditional methods with the purpose of evaluating its benefits and challenges in the teaching-learning process. Additionally, simultaneous implementation of GeoGebra *software could be explored* in groups addressing both mathematics and physics learning units, with coordinated activities. This would allow the dynamic features of the *software* to be more widely highlighted and encourage collaboration between students with different levels of experience using GeoGebra.

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