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Artículos científicos

Construcción del sujeto matemático: sinergia en la enseñanza de las matemáticas

Construction of the mathematical subject: synergy in mathematics education

Construção da disciplina matemática: sinergia no ensino de matemática

Sonia Janeth Hernández Carrillo
Sistema Estatal de Telesecundaria Durango, México
soniajanethhernandezcarrillo@gmail.com
https://orcid.org/0009-0009-0052-3331

Resumen

En este ensayo se plantea la posibilidad de la sinergia entre las teorías de las condiciones para el aprendizaje de Gagné (1987) y la objetivación de Radford (2014, 2023) en el contexto de la enseñanza de las matemáticas. Para ello, se analiza cómo la objetivación, subjetivación y la labor de la teoría de Radford pueden complementar los eventos para la enseñanza propuestos por Gagné, tales como generar atención, informar el objetivo de aprendizaje, estimular el recuerdo previo, proporcionar orientación, evocar el desempeño, dar retroalimentación, evaluar el desempeño e incrementar la retención y generalización con el fin de promover un aprendizaje significativo y profundo. Además, se destaca la importancia de considerar tanto los aspectos cognitivos como los socioepistemológicos en el diseño de estrategias matemáticas y se proponen eventos en la enseñanza que resultan de la complementariedad de ambas teorías. Se concluye con una reflexión sobre la relevancia y el potencial de esta sinergia en el campo de la educación matemática, y se sugieren direcciones futuras de investigación para continuar explorando esta perspectiva integrada en la enseñanza de las matemáticas.

Palabras clave: aprendizaje activo, educación básica, formación, instrucción, práctica pedagógica.

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Abstract

This essay explores the potential synergy between Gagné's (1987) theories of conditions for learning and Radford's (2014, 2023) objectification within the context of mathematics education. It analyzes how Radford's theories of objectification, subjectivation, and joint work can complement Gagné's proposed teaching events, such as capturing attention, stating learning objectives, stimulating prior recall, providing guidance, eliciting performance, offering feedback, assessing performance, and enhancing retention and generalization, all aimed at promoting meaningful and deep learning. Furthermore, the significance of considering both cognitive and socioepistemological aspects in designing mathematical strategies is emphasized, and teaching events resulting from the synergy of both theories are proposed. The essay concludes with a reflection on the relevance and potential of this synergy in the field of mathematics education, along with suggesting future research directions to further explore this integrated perspective in mathematical instruction.

Keywords: active learning, basic education, training, instruction, pedagogical practice.

Resumo

Este ensaio levanta a possibilidade de sinergia entre as teorias de Gagné (1987) sobre as condições de aprendizagem e a objetivação de Radford (2014, 2023) no contexto do ensino de matemática. Para isso, analisa-se como a objetivação, a subjetivação e o trabalho da teoria de Radford podem complementar os eventos de ensino propostos por Gagné, como gerar atenção, informar o objetivo de aprendizagem, estimular a memória prévia, fornecer orientação, evocar o desempenho, dar feedback, avaliar o desempenho e aumentar a retenção e generalização, a fim de promover uma aprendizagem significativa e profunda. Além disso, destaca-se a importância de considerar aspectos cognitivos e sócio-epistemológicos no desenho de estratégias matemáticas e propõem-se eventos de ensino que resultam da complementaridade de ambas as teorias. Conclui com uma reflexão sobre a relevância e o potencial desta sinergia no campo da educação matemática, e são sugeridas direções de investigação futuras para continuar a explorar esta perspetiva integrada no ensino da matemática.

Palavras-chave: aprendizagem ativa, educação básica, formação, instrução, prática pedagógica.

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Introduction

Teaching mathematics involves a complex process that goes beyond the simple transmission of knowledge. To achieve a deep understanding in this discipline, it is essential to understand how students learn and create effective conditions for learning. In this context, the perspectives of Gagné (1987) and Radford (2014, 2023) offer enriching and complementary approaches to mathematics teaching, since the strengths of one theory, as will be analyzed later, complement the limitations of the other.

Gagné's instructional design model highlights the importance of various aspects in the learning process, such as attention, perception, information acquisition, retention, retrieval, and knowledge transfer. It focuses on how students process information and how teaching can be structured through a proposal of specific events to optimize their ability to retain and apply what they have learned.

On the other hand, Radford focuses on the social and cultural construction of mathematical knowledge, which is why he highlights the importance of objectification and subjectivation as complementary processes that are intertwined in the formation of the mathematical subject. This subject not only acquires mathematical knowledge, but also reflects on it and applies it in real contexts (Radford, 2006a, 2014, 2023).

For this reason, this essay will explore how the integration of the key aspects of Radford's theory of objectification in the teaching events proposed by Gagné contributes to the formation of an active, reflective and contextualized mathematical subject. The purposes of this article are to analyze the theories of Gagné and Radford, highlight their possible synergy and reflect on their relevance and potential in the field of mathematics education. The thesis proposed is that the integration of these perspectives in teaching practice can promote deeper and more meaningful mathematical learning for students, which would foster their understanding, applicability and appreciation of mathematics.

Development

Mentalistic learning theories

Mentalistic learning theories play a fundamental role in the field of pedagogical practices, since they focus on the perspective that the process of acquiring knowledge is intrinsically individual, characterized by a defined beginning and end, and differentiated from the activities routine. This approach influences the configuration of teaching by directing it towards capturing the students' attention in relation to the teacher and the proposed activities.

However, this orientation can have the side effect of isolating students from their external environment. Furthermore, these theories often support individualistic evaluation methods and demonstrations lacking context, and may sometimes view collaboration as cheating (Wenger, 2001).

In this sense, the predominant mentalistic theories in pedagogical practices, although they differ in their approaches, principles, and conceptualization of the subject, consider the individual as the subject of the learning process. In tables 1 and 2, you can observe some of their characteristics related to those expressed by Lave and Packer (2011).

Table 1. Exploring mentalistic theories

Psychological theories	Characteristics	Subject	Subject-world relationship
Behaviorist	Emphasizes conditioning and reinforcement through environmental control.	The individual is a passive recipient of stimuli from the world.	The relationship is unidirectional, since the individual receives stimuli from the world and responds without considering internal mental processes.
Cognitivist	It focuses on mental processes, such as structuring, organizing, and processing information.	The individual is an active processor of information.	The relationship is bidirectional, since the individual processes information from the world through his or her internal cognitive processes.

Table 2. Exploring mentalistic theories (continued)

Psychological theories	Characteristics	Subject	Subject-world relationship
constructivist	It emphasizes social interaction and the active participation of the student in the construction of their own learning.	The individual is an active builder.	The relationship is multidirectional. The individual constructs his knowledge and understanding of the world through interaction, and can influence his environment during this process.

Source: Own elaboration based on information collected by Lave and Packer (2011)

It is crucial to highlight that, despite their presentation as independent categories, each theoretical approach offers a unique perspective on the learning process and has influenced each other significantly. In the context of behaviorism, the importance of shaping student behavior through control of the environment is emphasized, with the teacher meticulously designing this environment to encourage desired behaviors.

On the other hand, the cognitivist approach places emphasis on the mental processes of the student, who is considered an active participant in his or her own learning. However, with the arrival of constructivism, the educational landscape is further expanded by focusing on cognitive processes, social interaction and the active participation of the student in the construction of their knowledge. Each approach provides valuable contributions to understanding learning development, although they also present their own strengths and limitations inherent to the theory.

Learning from the perspective of social cognition

Social cognition is a field of study that focuses on the process by which individuals perceive, recognize, and evaluate social events to construct a representation of the interaction environment (Valdivieso, 2010). The social cognition perspective highlights the importance of social and cultural interaction in learning.

From the perspective of social cognition, learning is understood as a social and active process of construction of meanings in a specific context of communication and interaction with others (Shotter , 2001).

Condor and An taki (2000) conceive learning as a discursive and social process of knowledge construction, which occurs through dialogue, interaction and negotiation of meanings with others in a specific context and culture.

According to Lave and Wenger (1991), learning is a social and situated process that occurs through participation in everyday practices in a specific environment, in which the participating subject is reconfigured and in turn reconfigures the world.

Perkins (1993) agrees with this consideration of learning, since he conceives it as a dynamic and distributed process that involves the interaction between the individual, his environment and his culture. This process is not limited to the acquisition of information, but involves the creation of meanings, the resolution of problems and the construction of new ways of thinking.

According to Wenger (2001), learning is a social and situated process that takes place in the context of communities of practice. This process involves the active participation and commitment of members in the community, as well as the acquisition of new knowledge, skills and the transformation of the individual's identity through their participation in the community. Communities of practice are characterized by sharing the same task and having common objectives and standards. In the learning process in communities of practice, the most experienced members act as mentors, transmitting their knowledge and skills to the more novice members (Rogoff, 1995).

Lozares (2000) agrees with Wenger and Rogoff, and states that learning is conceived as a social and situated process in which knowledge is acquired and developed through active participation in specific social and cultural situations. This process involves interaction and collaboration with other individuals, critical reflection on lived experiences and the construction of new knowledge through problem solving.

From the perspective of social cognition, the relevance of interaction, collaboration and critical reflection in education is emphasized, since it is highlighted that the mind is constructed collaboratively in various contexts and cultures.

> Considering what was previously stated, the coexistence of several theoretical approaches in learning and cognition is evident. Two notable perspectives are mentalistic theories and social cognition. While the former focus on internal cognitive processes, social cognition explores how social interactions and the cultural environment influence the processing and use of information (Uribe, 2010).

While mentalistic theories highlight essential cognitive mechanisms, they often neglect social and cultural context. In contrast, social cognition recognizes the influence of social factors on the acquisition of knowledge, emphasizing the relevance of communities of practice and the environment in the construction of knowledge. However, it can overlook certain individual aspects by focusing excessively on the social.

Therefore, this essay proposes to complement both perspectives in order to design mathematical instruction of greater significance. The synergy is based on the fusion of Gagné's (1987) theories of learning conditions, aligned with mentalistic theories and Radford's (2014, 2023) theory of objectification, which connects with social cognition. This would allow both individual and social aspects to be addressed, which would enrich the teaching of mathematics to achieve a deeper and more effective understanding.

Theory of conditions for learning by Robert Gagné

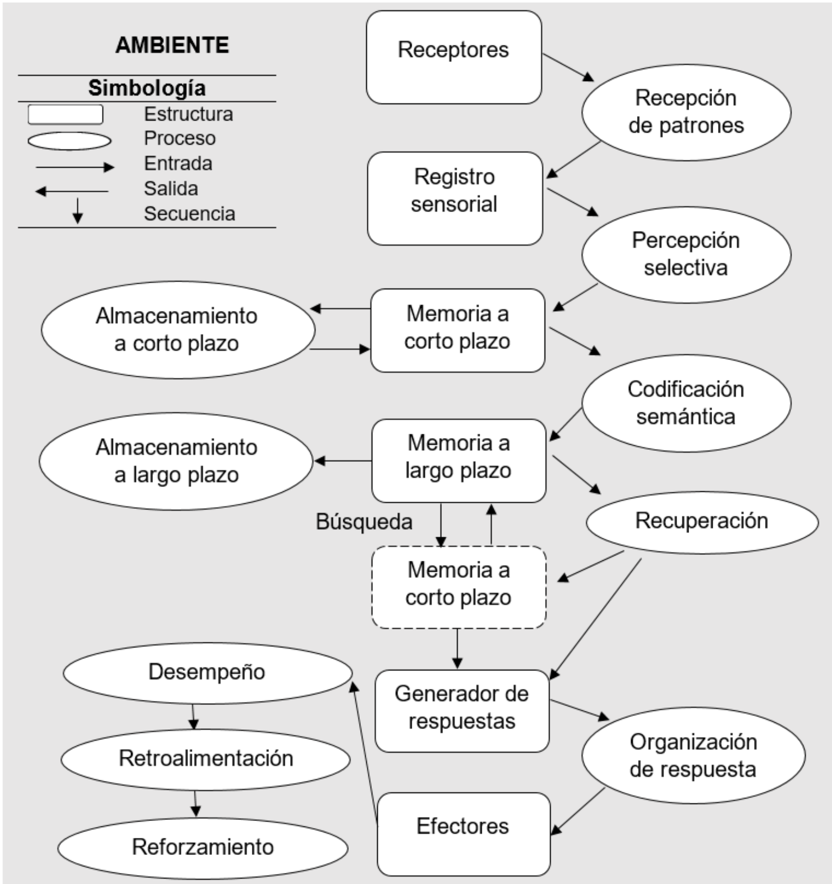
Gagné's (1987) theory of the conditions for learning is considered eclectic, since it serves as a bridge between behaviorism and cognitivism (Gottberg *et al.* , 2012). This theory focuses on the relationship between processing and learning, and seeks to answer the general question of what learning is.

According to Gagné (1987), learning involves "a lasting change in a human capacity or disposition that cannot be attributed to maturation, and can be evaluated by comparing behavior before and after the learning situation" (p. 2). In this sense, learning is the process of the cumulative effects of discrimination, generalization and transfer (Meza, 1993).

In Gagné's learning theory, instructional design, varieties of learning, conditions for learning, information processing, and events in instruction are identified as principles. Next, only the last two will be addressed, since they are considered the most basic principles to present the synergy analysis.

Gagné (1987) proposes an information processing model to explain how learning takes place, which consists of several stages and processes. As shown in Figure 1, information is received through the senses and processed in the sensory register. If the information is relevant, it is converted to neural information and stored in short-term memory (STM) . The capacity of this memory is limited and it can store information acoustically or articulately. The information is then semantically encoded and stored in long-term memory (LTM) permanently, although it may be difficult to access due to interference or forgetting. To recover the information stored in the MLP, tracks or indexes are required that facilitate the search and recovery process. The recovered material can be processed again in the MCP or used directly in the generation of automated responses or skills. The memory of previously learned information can manifest itself shortly after learning or at later times.

Figure 1. Learning and memory processes



Source: Adaptation of Gagné's information processing model [Diagram] (1987). *In the conditions of learning* (p. 77). Inter-American.

To begin the learning process, it is crucial to capture the student's attention and generate expectations, which can be achieved through strategies that adjust the environment, such as varying the tone of voice and presenting clear learning objectives. Subsequently, we proceed to recover the information pertinent to the topic of the MLP, stimulating memory from the MCP using clues.

Furthermore, selective perception plays a fundamental role in learning, where the student must be able to discern and process relevant information. Therefore, the teacher must present the material in an organized manner to facilitate this process.

Semantic coding constitutes another crucial process, transforming information into a meaningful format for the student. Here, it is essential that the teacher provides guidance to help students deeply understand and encode information.

Once the information has been internalized in the MLP, the student issues responses. To encourage this event, the teacher must offer opportunities for students to apply and practice what they have learned. Table 3 highlights more relevant events in both processes, such as reinforcement, performance evaluation, recovery and generalization, which contribute to the consolidation of learning and the increase in retention and generalization of the acquired knowledge.

Table 3. Correspondence between events in the learning and teaching process

Event in learning	Event in teaching
Reception	Generate attention
Expectation	Inform the learning objective
Recovery to the MCP	Stimulation of memory (prior knowledge)
Selective perception	Present the stimulus material
Semantic coding: entry to the MLP	Provide learning guidance
Response broadcast	Evoke performance
Reinforcement	Give feedback
Recovery and reinforcement	Evaluate performance
Recovery and generalization	Increase retention and generalization

Source: Adaptation of relationships between learning phases and teaching events (Gagné, 1987, p. 313). *Inter-American.*

According to recent research, teaching planning from Gagné's perspective has proven to be beneficial for learning. For example, Noroña *et al.* (2016) state that designing classes with this approach can increase students' interest in the topic and encourage their engagement in individual and group activities. Likewise, it can help develop intellectual skills and improve your academic performance by working under this methodology.

Objectification theory

In the field of mathematics education, the theory of objectification developed by Radford (2006a, 2006b, 2014, 2023) advocates that teaching and learning processes focus on the formation of ethical and reflective individuals, capable of critically positioning themselves in historical and cultural mathematical practices. From this perspective, mathematics education is conceived as a political, social, historical and cultural effort that seeks to form individuals committed to reflection and ethics in relation to mathematical practices (Radford, 2014), which highlights the importance of considering the sociocultural context in mathematics teaching.

In this vision, teaching and learning converge in the same activity in which students are active participants who interact and question the content and relate it to their previous experiences. Communication and collaboration emerge as essential elements in this process, which contributes to the collective construction of knowledge (Radford and Mendonça , 2022).

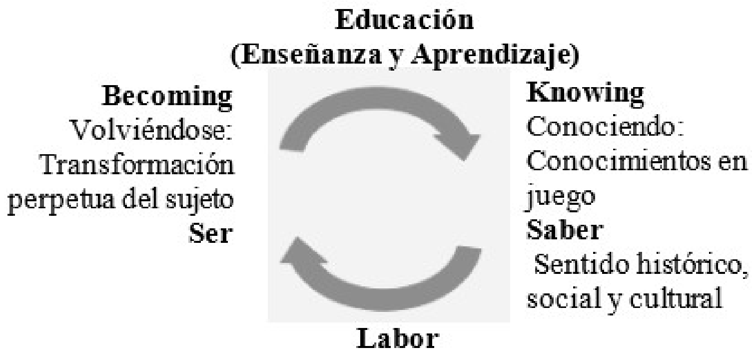
The theory of objectification is based on a political and conceptual position that recognizes that education not only involves the transmission of knowledge, but also the development of subjectivities (Vergel, 2021).

This theoretical framework incorporates central principles, such as the idea of joint work, semiotic mediation, objectification and subjectivation. Firstly, during joint work, an encounter is established between the Other and the world, which causes a space in which "individuals create culture and culture occurs in the work or work" (Radford, 2014, p. 137). Thus, teaching and learning are understood as processes contextualized in specific practices, influencing the construction of mathematical knowledge and the attribution of meanings through the understanding of mathematical semiotics (Radford, 2006b).

In this context, knowledge is conceived as a possibility that takes shape in practice, as exemplified in figure 2. The dynamics of learning and teaching are understood from a dialectical conception as follows:

> The expression of a way of life: a joint work that occurs in a socio-political space within which knowing *(* "Knowing") and *becoming* ("Becoming") take place, that is, becoming a subject as a historical project. social always unfinished, always in motion (Radford, 2014, p. 138) .

Figure 2. Principles of objectification theory



Source: Radford adaptation(2014, p. 139)

In this perspective, the concepts of objectification and subjectification are linked to the processes of knowing and transforming into someone or something. At birth, individuals face a world full of meanings that have been objectified by previous generations, through the creation of signs and symbols that can be shared and understood (Radford, 2006b). Objectification leads to awareness of that world independent of individuals, with rules, meanings, properties and structures that are transmitted through language. Thus, objectification is defined as “the social, corporeal, and symbolically mediated process of becoming aware of and critically discerning historically and culturally constituted forms of expression, action, and reflection” (Radford, 2014, p. 141) .

On the other hand, when the subject appropriates that objectified world, transforms it and makes it his or her own, the process that happens is subjectivation. In this, the subject attributes meaning and personal meaning to mathematical objects, relates them to their previous experiences and integrates them into their own identity and subjectivity. In this way, subjectification “consists of those processes through which subjects take a position in cultural practices and are formed as unique historical cultural subjects. Subjectivation is the historical process of creation of the self” (Radford, 2014, p. 142).

Regarding the teaching-learning activity, Radford indicates that they are processes that involve the progressive transformation of cultural knowledge into an object of consciousness through collaboration between teachers and students. The materialization of knowledge is achieved through the use of signs, diagrams, language and other semiotic forms that allow its recognition and reflection (Radford, 2006b, 2023).

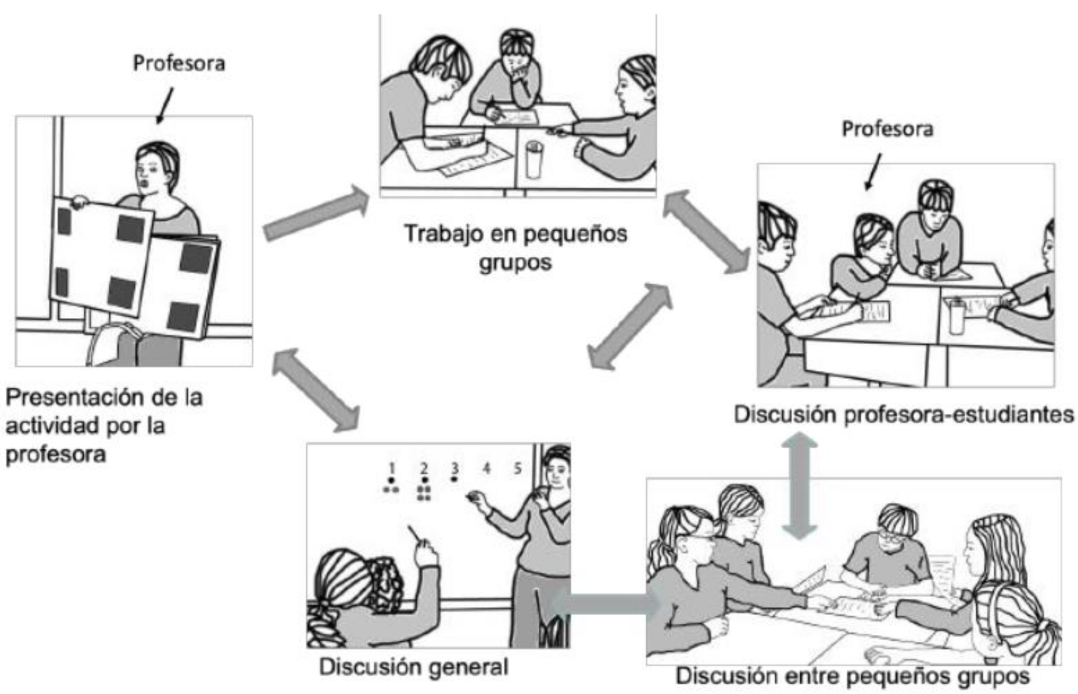
Following the above, the teacher's purpose is to offer students an alternative way of approaching mathematical problems, which are considered from a historically and culturally constituted approach. This perspective can be “refuted, which is part of the collective co-positioning of voices and subjectivities of both the students and the teacher" (Radford, 2023, p. 248).

In this way, Radford (2023) indicates that the teaching-learning activity consists of two main components: the didactic organization (or Φ structure) and the activity itself (joint work). The Φ structure refers to the planning and structuring of the educational process by the teacher, where the objectives, goals and specific tasks are established, which are designed with increasing difficulty.

The objectification process occurs when students and teachers, working together, materialize the knowledge to which the didactic project is directed, that is, they turn it into something conscious. Through this materialization, students begin to become aware of that knowledge. As can be seen in figure 3, Radford (2023) proposes three moments in the teaching and learning activity:

1. The teacher presents the activity and students work in small groups while the teacher visits those groups and asks questions or comments.
2. The teacher can invite two or more groups to meet to discuss a specific idea.
3. At some point, the teacher may invite the entire class to participate in a general discussion, where groups present their ideas and other groups can critically question them or make suggestions for improvement or generalization.

Figure 3. The activity Φ and some of its moments



Source: Taken from the activity Φ and some of its moments (Radford, 2023, p. 112)

In the research field, Radford's (2006a, 2014, 2023) objectification theory improves mathematics teaching. Godino *et al* . (2007) propose an ontosemiotic approach based on this theory to achieve a deeper understanding of mathematical knowledge in classrooms. This approach is based on an interaction between mathematical objects and students through semiotic communication. Thus, difficulties in understanding complex concepts are identified and strategies to overcome them are designed.

Synergy between the theories of Gagné and Radford

In the previous segments, a detailed analysis of the theories developed by Gagné (1987) and Radford (2006a, 2014, 2023) has been provided. At this point, it is essential to highlight the similarities and differences inherent in these theoretical perspectives to identify their strengths and limitations, which will pave the way to propose a synergy between both theories.

Both approaches recognize the importance of motivation in the learning process and highlight the generation of interest and the establishment of clear goals and objectives for students. Furthermore, both perspectives point out the importance of presenting the content gradually and progressively, considering the level of complexity and the students' prior knowledge. They also emphasize the need to provide effective feedback and conduct periodic assessments to confirm learning and promote continued growth.

Despite these convergences, however, there are notable differences between the two theories. For example, Gagné focuses on the student's internal cognitive processes, while Radford adopts a socioepistemological approach that highlights social interaction and the co-construction of knowledge. Gagné incorporates behaviorist influences in his approach, while Radford moves away from this perspective and emphasizes joint work, where knowledge already exists and is objectified and subjective through the interaction between participants. Additionally, Gagné focuses on instructional design generally, applicable to various disciplines, while Radford focuses specifically on mathematics teaching. Radford also emphasizes the importance of corporeality, touch, and human perception in the construction of mathematical meanings, aspects that Gagné does not explicitly address.

Table 4 shows the strengths and limitations of Radford and Gagné's theories that influence the way teaching and learning processes are approached.

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Table 4. Strengths and limitations of Radford and Gagné's theories

Theory	Strengths	Limitations
Objectification	Emphasizes the collective construction of knowledge and social interaction. Encourages collaboration between teachers and students. Recognizes the relevance of subjectivity and reflexivity in learning.	It lacks a clear guide to sequence and structure content. May not adequately consider individual needs.
The conditions for learning	It offers a structured approach to the design and sequencing of instruction. Highlights the importance of student attention and motivation.	Approach with a tendency towards behaviorism, which limits the consideration of socio-cognitive and emotional aspects of learning. Little attention to the social and cultural contexts that influence the learning process

Source: self made

As can be seen in the table above, the limitations identified in one theory contrast with the strengths of the other. In other words, the synergy between Radford and Gagné's theories is based on the effective integration of their key elements. By combining the learning events proposed by Gagné with Radford's concepts of objectification, subjectification and joint work, a more complete and effective perspective is achieved for designing educational experiences in the mathematical field.

The relevance of this synergy lies in its ability to address various essential aspects in education. As presented in Tables 5 to 7, by focusing on generating interest and personalized motivation, an energetic and sustainable start to the learning process is ensured. Clear communication of lesson objectives and their connection to students' daily lives instills a sense of purpose, which can increase engagement and involvement from the start.

In the event of stimulating previous memory, the joint work proposed by Radford adds a new level of enrichment. Guiding students to reflect on their prior knowledge and discuss it in groups before introducing new content not only activates memory, but also promotes the co-creation of connections and meanings, thereby establishing a solid foundation for the assimilation of new concepts. .

The subjectivation inherent in Radford's theory plays a crucial role in several events. For example, reporting learning objectives and linking them to students' personal experiences and perspectives can increase their motivation and understanding. Similarly, in the guidance phase, allowing students to explore diverse problem-solving strategies enriches their thinking by promoting diversity of approaches and demonstrating that there is no single right path.

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The social interaction proposed by Radford is essential in events such as evoking performance and giving feedback. Collaborative work among students in problem solving and peer feedback enrich understanding and improve the solution process, which promotes collective learning.

In the event of evaluating performance, subjectivation becomes important by allowing students to self-evaluate and justify their solutions. This approach encourages deeper understanding and metacognition, as students must reflect on their own thinking and learning process.

Finally, in the event of increasing retention and generalization, objectification can manifest itself in the creation of visual or tangible representations that summarize key concepts. These representations can serve as physical references to reinforce knowledge retention and transfer.

Table 5. Synergy: Radford and Gagné (RG)

Process in learning-event in teaching (Gagné, 1987)	Explanation of the teaching event in RG synergy	Event on teaching at RG synergy
Reception-generate attention	The teaching design should include elements that capture students' attention, such as interesting introductions, challenging problems, or interactive activities. This promotes motivation and commitment to learning.	Generate initial interest: Start the class with a challenging question or an interesting introduction that captures students' attention and awakens their curiosity about the topic that will be addressed.
Expectation-inform learning objective	It is essential to clearly and explicitly communicate the objectives of teaching. In addition, emphasis is placed on relating learning objectives to the daily reality of students to create expectations and motivate their interest.	Establish clear objectives: Explain the lesson objectives clearly and concisely, highlighting their relevance to students' daily lives, to motivate their interest in the content.
Recovery to MCP-stimulation of memory (prior knowledge)	Before introducing new concepts, it is important to activate students' prior knowledge related to the topic. This is achieved through the collective construction of shared knowledge in the joint work proposed by Radford.	Activate prior knowledge: Carry out a group activity where students share their prior knowledge about the topic and relate it to the new information that is going to be presented.

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Table 6. Synergy: Radford and Gagné (RG)

Process in learning-event in teaching (Gagné, 1987)	Explanation of the teaching event in RG synergy	Event on teaching at RG synergy
Selective perception - presenting stimulus material	In this stage, mathematical content is introduced in a clear, structured and contextualized way, to improve the selective perception and relevance of the information. Different resources can be used, such as examples, visual demonstrations, practical exercises and concrete applications. This is where the Φ structure of Radford's teaching-learning activity becomes important, as specific objectives, goals and tasks are established.	Present concrete examples: Present an explanatory video that shows concrete examples and practical applications of the mathematical concepts to be discussed. In addition, use practical exercises and visual demonstrations that allow students to perceive the importance and usefulness of the concepts in real situations.
Semantic coding: entry into MLP-providing learning guidance	Provide learning guidance during this process, they are presented with strategies and guided in the construction of knowledge and work collaboratively to facilitate semantic encoding and entry into the MLP. This relates to the semiotic nodes and objectification proposed by Radford, where collaboration is promoted in the creation of mathematical interpretations.	Promote collaborative work: Divide students into small groups and provide them with a step-by-step guide to solving problems related to the topic. Encourage discussion and exchange of ideas among group members to collaboratively build a solid mathematical interpretation.
Response emission - evoke performance	Collective construction of knowledge to facilitate the evocation of responses by students and promote their active participation. Students are asked to put into practice what they have learned by solving problems or applying concepts in concrete situations. This allows them to develop skills and transfer knowledge to different contexts. At this point, joint work and Radford's three-moment approach come into play, as group work and discussion of ideas are encouraged.	Apply the concepts learned: Perform a problem-solving activity in groups where students apply the concepts learned to find solutions. At the end, ask them to share their answers and justify their reasoning, promoting active participation and the collective construction of knowledge.

Table 7. Synergy: Radford and Gagné (RG)

Process in learning-event in teaching (Gagné, 1987)	Explanation of the teaching event in RG synergy	Event on teaching at RG synergy
Reinforcement-giving feedback	Feedback and the collective construction of knowledge are essential for effective reinforcement of learning and improvement of performance. Critical reflection and the collective co-positioning of voices and subjectivities are promoted.	Provide feedback: Give individual and group feedback on the results of the problem-solving activity. Highlight successes and suggest improvements, encouraging critical reflection and the exchange of opinions among students to enrich collective learning.
Recovery and reinforcement-evaluate performance	Performance evaluation is carried out through the collective construction of knowledge, facilitating the recovery and reinforcement of the acquired knowledge. This encourages the formation of ethical and reflective subjects about mathematical practices.	Assess learning: Conduct an evaluation that includes questions that require the application of concepts learned in new contexts and critical reflection on the usefulness and ethical implications of mathematical practices. Provide constructive feedback to strengthen learning and promote the formation of ethical and reflective subjects.
Retrieval and generalization-increase retention and generalization	To consolidate learning and encourage the transfer of knowledge to new situations, additional opportunities for practice, review and application are offered. This is related to the increasing difficulty in problems proposed by Radford, where students gradually advance towards more complex levels of conceptualization.	Encourage knowledge transfer: Provide additional practice activities that present new challenges and require the transfer of knowledge to different situations. Gradually increase the difficulty of problems so that students develop more complex conceptualization skills and achieve deeper learning.

Source : Own elaboration

The synergy of Radford and Gagné's theories creates a comprehensive pedagogical approach that responds to the complex demands of the learning process. By considering and leveraging the strengths of each theory while mitigating its limitations, a more effective and enriching instructional design is achieved. This integration not only involves the teaching events proposed by Gagné, but also benefits from the subjective and social perspectives proposed by Radford, generating a deeper and more meaningful educational experience.

Conclusions

The synergy between the theories of Gagné and Radford is not only a pedagogical approach, but also an enriching and promising perspective in the field of mathematics education. The strategic integration of key elements of both theories has resulted in a solid and comprehensive pedagogical framework that addresses various dimensions of the teaching and learning process. This strategic convergence could expand the effectiveness of educational practices, in addition to comprehensively responding to the complex needs of students in their quest to master mathematics.

The relevance of this synergy is manifested in its ability to address multiple crucial aspects of teaching practice. Generating interest and motivation at the beginning of lessons establishes a solid foundation for continued learning. Clear and transparent communication of objectives, along with linking mathematical concepts to students' everyday reality, instills a sense of purpose and relevance, which in turn can increase involvement and commitment.

Synergy also underlines the importance of social interaction and co-construction of knowledge. By encouraging active collaboration and co-construction of meaning, opportunities are created for deep understanding and critical exploration of mathematical concepts. Constant, constructive feedback further reinforces the learning process, allowing for continuous improvement and solid internalization of content.

One of the most exciting aspects of this synergy is its transformative potential. The consolidation of learning through practice and the transfer of knowledge represents a fundamental pillar. The gradual progression of challenges enables students to apply concepts in diverse contexts and develop flexible and analytical thinking skills. The attention to the collective construction of knowledge and joint work proposed by Radford integrate perfectly with Gagné's principles of instructional design and information processing, which enriches the learning experience.

Within the field of mathematics education, this synergy takes on special relevance. The often abstract and challenging nature of mathematics demands pedagogical approaches that address both the cognitive and social aspects of learning. The integration of Gagné and Radford's approaches provides a platform that meets these needs while promoting a deep and contextualized understanding of mathematical concepts.

For all of the above, it can be concluded that this essay has presented a solid proposal, so, in the research field, it would be valuable to carry out exhaustive empirical research to evaluate the effectiveness of this synergy in real educational contexts and at different levels.

educational. Furthermore, exploring how this integrated perspective could be adapted to other disciplines and how it would specifically address cultural and gender barriers in mathematics education could open up an intriguing and potentially transformative field of research. Ultimately, the synergy between Gagné and Radford's theories not only enriches mathematical pedagogy, but can also influence the way education as a whole is conceptualized and approached.

References

Condor, S. and Antaki, C. (2000). Social cognition and discourse . In TA van Dijk (ed.), *Discourse as structure and process* (pp. 453-489). Gedisa. https://padron.entretemas.com.ve/cursos/AdelD/unidad3/12CognicionSociaDiscurso.pdf

Gagné, R. M. (1987). The conditions of learning. Inter-American.

Godino, JD, Batanero, C. and Font, V. (2007). The onto- semiotic approaches to research in mathematics education. ZDM, 39, 127-135. https://doi.org/10.1007/s11858-006-0004-1

Gottberg , E., Noguera, G., and Noguera, M. (2012). Learning seen from the eclectic perspective of Robert Gagné and the use of new technologies in higher education. Universities, (53), 50-56. http://www.redalyc.org/articulo.oa?id=37331092005

Lave, J. and Wenger, E. (1991). Located learning . Legitimate peripheral participation . Cambridge University Press .

Lave, J. and Packer, M. (2011). Towards a social ontology of learning. Journal of Social Studies, 40, 12-22. http://www.scielo.org.co/scielo.php?script=sci_arttext&pid=S0123-885X2011000300002

Lozares , C. (2000). Situated activity and/or socially distributed knowledge. Papers , 62, 97-131. https://papers.uab.cat/article/view/v62-lozares

Meza, AL (1993). Epistemological analysis of Robert Gagné's cumulative learning model. Psychology, 11(2). https://doi.org/10.37843/rted.v1i1.260

Noroña, MA, Flores, CB and Flores, FF (2016). Influence of the application of Robert Gagné's learning theory on academic performance, in the study of the Oersted Experiment. Latin American Journal of Physics Education, 10(1). http://www.lajpe.org

Perkins, D. (1993). The most distributed person of thinking and learning. Paidós.

Radford, L. (2006a). Elements of a cultural theory of objectification. Latin American Journal of Research in Educational Mathematics, 103-129. http://www.redalyc.org/articulo.oa?id=33509906

Radford, L. (2006b). Semiotics and mathematics education : An introduction. Relime, 7-21. https://www.researchgate.net/publication/28132137

Radford, L. (2014). From the theory of objectification. Latin American Journal of Ethnomathematics, 7(2), 132-150. http://www.redalyc.org/articulo.oa?id=274031870010

Radford, L. (2023). The theory of objectification: a Vygotskian perspective on knowing and becoming in the teaching and learning of mathematics. Nomos.

Radford, L. and Mendonça , T. (producers). (April 22, 2022). V. EPEM: an introduction to ethnomathematics (video). Youtube. https://www.youtube.com/watch?v=qOyunsMg1HM

Rogoff, B. (1995). The three plans of cultural activity : Appropriative participation , guided participation , and apprenticeship . In J. Wertsch, P. del Río, and A. Álvarez (eds.), Sociocultural studies of mind (pp. 139-164). Cambridge University Press .

Shotter , J. (2001). Conversational realities. The construction of life through language. Amorrortu.

Uribe , C. (2010). A brief introduction to social cognition: process and related structures. Contexts, 4, 33-40. http://www.contextos-revista.com.co/Revista%204/A5_Una%20introduccion%20a%20la%20cognicion%20social.pdf

Vergel, R. (October 23, 2021). An approach to the theory of objectification (video). Youtube. https://www.youtube.com/watch?v=oVDQxwry5AM

Wenger, E. (2001). Communities of practice. Learning, meaning and identity. Paidós.