El método del aprendizaje basado en problemas como una herramienta para la enseñanza de las matemáticas

The method of problem-based learning as a tool for teaching mathematics

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Resumen

El Aprendizaje Basado en Problemas (ABP) es una de las metodologías educativas que han tenido buena aceptación en instituciones universitarias en México. Es un proceso activo de aprendizaje que funciona mediante la solución de problemas relacionados con la interacción de los estudiantes y su entorno profesional. La esencia del ABP consiste en identificar, describir, analizar y resolver tales problemas, lo cual se logra con ayuda del docente, desempeñando así otro papel tanto el proceso de enseñanza-aprendizaje como los estudiantes.

Una forma en la que se ha incorporado recientemente esta metodología en el salón de clases de las carreras de ingeniería ha sido mediante trabajos teóricos que los estudiantes discuten. Por ejemplo, discuten los resultados de un artículo científico reciente en particular, del cual deben analizar los planteamientos presentados y que están relacionados con sus cursos de matemáticas impartidos en la institución. Los objetivos principales son implementar el ABP en los cursos del área de matemáticas como parte de las metodologías que los docentes tienen a su alcance e involucrar a los estudiantes en investigaciones de ciencia e ingeniería de materiales; la meta principal es que los estudiantes presenten sus resultados tanto en foros
estudiantiles como en congresos nacionales, como producto de la aplicación del ABP en el salón de clases.

**Palabras clave:** Aprendizaje Basado en Problemas (ABP), ciencia e ingeniería de materiales.

**Abstract**
Problem Based Learning (PBL) is one of the educational methodologies that have been well accepted in universities in Mexico. It is an active learning process that works by solving problems related to the interaction of students and their professional environment. The essence of PBL is to identify, describe, analyze and solve such problems, which is achieved with the help of teachers and other role playing both the teaching-learning process as students.

One way that has recently joined this methodology in the classroom engineering careers has been theoretical work by students discuss. For example, discuss the results of a recent scientific article in particular which should analyze the proposals submitted and that are related to their math courses taught in the institution. The main objectives are to implement PBL in the area of mathematics courses as part of the methodologies that teachers have to reach and engage students in science and engineering research materials; the main goal is for students to present their findings both in student forums and national conferences, as a result of the application of PBL in the classroom.

**Key words:** Problem Based Learning (PBL), materials science and engineering.

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**Introduction**

The objectives of this work are: a) that students learn to solve engineering problems through the systematic search for information and scientific reasoning, developing issues not contained in the programs of mathematics courses, using this method Based Learning Troubled (ABP), and b) that the teacher becomes a transmitter of new knowledge, as are the results of recent
research, to motivate students with issues of current engineering but without neglecting the theory from which they were developed.

PBL is an active learning process that works through the solution of problems associated with interaction of man and his environment. The essence of PBL is to identify, describe, analyze and solve such problems, which is achieved by the interaction of teachers and students.

The goals to be achieved by the ABP is that the student:

1. responsible for self-learning is made; diagnosis you need to know about a particular problem.

2. Encourage scientific reasoning from the formulation of hypotheses to the systematic search for solutions to specific problems.

3. Work harmoniously with peers through good communication, be available to help their peers, develop different roles, listen and have confidence to make its best effort, that is, you make a group construct.

4. Know the progress of the current program, the learning process of problem solving and knowledge acquisition; is capable of self-assessed.

This is thus an attempt, which has been established in other instances, such as the European Community to establish the institution in the spirit of learning to learn. (1) Once established practice, the next step is to implement the pedagogical basis in student training: learning to be, learning to do, learning to know and learning to live collectively. This will ensure that students acquire the skills necessary for the core competencies of their profession. (2)

Special emphasis is placed on the teacher-student relationship, which is intensely interpersonal looking for students to learn from the teacher. (3)
In practice, the student must learn the topics of their courses through research projects grounded in disciplines of knowledge related to their profession.

It is intended that students self-assess their learning, sustained in a self-posture, effective search of information and cooperative work within a team, among many other aspects that help meet the demands of society. (4)

THE COURSE
PBL methodology was implemented in an intermediate course in the curriculum, so that students with the necessary theoretical basis to develop the themes of the course after having passed their basic courses, while their specialization courses start having assimilated this new methodology. During Differential Equations was selected because it is a matter that requires use prior knowledge of their courses Differential and Integral Calculus and Linear Algebra, which in turn will be useful in other courses in the area of engineering such as circuits Electric, Electricity and Magnetism, among others.

A typical course of Differential Equations covers, among other topics shown below:

- First order differential equations
- Applications of first order differential equations
- Linear differential equations of higher order
- Applications of differential equations of second order

Among these issues, he was chosen applications of differential equations of second order to be of great interest in the area of engineering. And as a sub in the PBL methodology polymers, polyelectrolytes specifically, was selected through a recent scientific article, which the teacher is co-author. (5)

The group raised the following goals in relation to the objectives of the university textbooks differential equations as for students is to step beyond:

- Solve second order differential equations which involved partial derivatives.
- Solve second order differential equations in different coordinate systems.

For its part, the teacher establishes three purposes in their educational evaluation:

- Proof of ownership or competition in a given area.
- Talk to the teacher and / or academia, on areas for improvement of the teaching-learning process.
- Discussion with students about the level of competition achieved.

For demonstrating mastery or competence in a certain area, students should be able to establish what the separation of variables, to express both the Laplacian operator in cylindrical coordinates and spherical coordinates, and finally solve the Poisson equation -Boltzmann to Zwitterions in spherical and cylindrical coordinates.

For discussion the teacher and / or the academy on areas where they can improve the process, students raised the updating of curricula and teachers who teach elementary math courses, a discussion was held at the beginning of semester of the issues in the curriculum and extra-class projects with real and current applications of the topics covered in the curriculum they were developed. Students expressed concern about the need for higher education institutions have researchers in the field of science teaching elementary courses in his career.

In discussion with students about the competence level achieved, the goal to present the work done in student meetings and / or forums at national level was raised.

Not being the assessment of student performance and knowledge exclusive responsibility of teachers, fellow students and the students themselves become evaluators and can produce only information that validates and complements teacher observations. Thus, the evaluation areas are the content, processes and the product achieved.
The content is evaluated according to the information and knowledge acquired by students, use tests "objective" true-false, multiple choice or short answer and are focused on the teacher. That is, it determines content, depth and level of understanding. The labor issues are ordinary differential equations (ODE), second order, industrial applications of second order ODE and partial EDO.

The process is evaluated based on the methods and techniques developed by the students, whom a particular item is ordered and intellectual. This type of evaluation focuses on the student's ability, as an apprentice, to structure a scheme to solve a problem, to use information to solve problems and evaluate the information or data collected. All this is done by exposing the subjects to the group.

The product of any activity can be used to evaluate any type of learning objectives. Particularly the presentation of the results was elected student forums.

THE HOMEWORK
A scientific article selected from students two tasks arise to argue, on the one hand it is the derivation of the Poisson-Boltzmann linearized, for the special case of zwitterions (PBLZ), and another is the solution of this equation in the cylindrical coordinate system. The goal is to analyze the parameter screen of a polymer system. (6)

The main difference between the two tasks is that while the first task focuses more on conceptual understanding, the second task requires a mathematical development which goes beyond mathematical methods. Two work teams are formed and discussed, without the intervention of the teacher, the assignment of tasks to each team.

Both teams start their respective task together with a literature search of some important for the development of theories related to the topic of polyelectrolytes, which had already been specified by the teacher events. See Table 1.
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<td>E. Fisher</td>
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Table 1. Relevant Events on polyelectrolyte

For the derivation of the Poisson-Boltzmann linearized, discussion of the task is initiated by exposure, by the teacher, the topics of electric field and potential, reaching up to the approach of the Poisson equation

\[
\nabla^2 \Psi = -\frac{4\pi \rho}{\varepsilon \varepsilon_0}
\]

(1)

Where \( \Psi \) It is the electric potential, \( \rho \) It is the distribution system load, \( \varepsilon \) It is the dielectric constant of the material and \( \varepsilon_0 \) It is the dielectric constant of the medium.

From this point, the teams start their work tasks and not being allowed students to interact with any other student teams.
Once exposed threads of electric field and potential students of the first team to get an expression, in linear terms, for the mean-field approximation of the charge distribution is requested.

\[
\rho = \sum_i n_i^0 z_i e \exp\left(-\frac{z_i e \Psi}{kT}\right)
\]  

(2)

In the above expression \( z_i \) It is the valence of the \( i \)-ésima ionic species, \( e \) It is the elementary charge, \( n_i^0 \) is the concentration of the species, \( k \) is the Boltzmann constant and \( T \) It is the temperature.

The expression that must be reached is the same to which the authors of Article arrive:

\[
\rho = -2n^0 z^2 e^2 \frac{\Psi}{kT}
\]  

(3)

This generalization to Zwitterions of the Debye-Huckel is obtained.

\[
\nabla^2 \Psi = \lambda^2_M \Psi
\]  

(4)

To level the workload of the two teams made it to the first team was asked also resolved PBLZ, Equation 4, in spherical coordinates, having had to come to find the interaction parameter \( \Psi_M \).
The second task of the ideal model of a coaxial cylinder, see figure 1, which contains a dielectric region, the cylindrical shells of radius a and b have uniform densities linear loads and of opposite sign, $\lambda$ y $-\lambda$, respectively. Finally, the length of the cylinder is taken too large to neglect the effects of extreme.

Students should express the potential function in cylindrical coordinates, as proposed by the authors of the article:

$$
\Psi = \Psi(\rho, \varphi)
$$

and then separated in terms of the variable of interest $\rho$ and $\varphi$, to obtain two second order differential equations by the method of separation of variables:

$$
\frac{d^2Q}{d\varphi^2} + n^2Q = 0
$$

Which the solution is $Q(\varphi) = A sen(n \varphi) + B cos(n \varphi)$
\[ \rho^2 \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} - n^2 = 0 \]  

(7)

With solution \( R(\rho) = \rho^n + \rho^{-n} \).

These two equations were discussed by the second team, however, their solutions were discussed by the entire group, not only for the members of the second team, as they are of great interest for the purposes of this study.

Once the discussion of the proposed solutions to the eq. (6) and (7), continues the task of the second team with the general solution in terms of \( \rho \) and \( \varphi \), proposed in Article.

\[ \Psi(\rho, \varphi) = \sum A_n\, \text{sen}(n\varphi)\left(C_n\rho^n + D_n\rho^{-n}\right) + D_0 \ln \left(\frac{\rho}{\rho_0}\right) \]

(8)

After completing the two tasks by the students, a discussion session in which the group talked about the achievements was held, as well as the methodology used.

Upon completion of the tasks made by both teams and the respective findings, the teacher, as a seminar, developed the solution with boundary conditions specified by the authors.

\[ \Psi(a, \varphi) = V_a \]  

(9a)

\[ \Psi(b, \varphi) = V_b \]  

(9b)

Where \( V_a \) y \( V_b \) are the potentials of the cylindrical shells.

From (8), (9a) and (9b), and after a little math, it finally reaches the expression for the coefficients of the general solution in terms of \( \varphi \), which show as follows:
Finally, the progress of this work by the students, under the guidance of teachers in the Third National Congress of Metallurgy and Materials, developed in the city of Monclova, Coahuila presented., From 28 to 30 September 2005 (7) and in which the students had the opportunity to interact with professional engineers, both in research and in industry.

CONCLUSIONS
PBL methodology was well received by engineering students. One of the points where all the students agreed was to change their role in the classroom, from totally passive, in which no mention was a virtue, an active attitude that to the depth with which establish rightful topics touching them. Another motivating points was considered the teacher took off his powerful aura, accepting both criticism and other possible solutions to problems in the classroom, so students more involved in the discussions.

For teachers this was an experience that could transmit knowledge to students beyond the course content, and involve the institution in such educational methodologies which are not yet accepted by all teachers.

The main problem they faced was how long the group, mainly because the total of the courses are still designed from direct teaching models and receptive learning, and the teaching strategy most commonly used is the lesson masterly. For that reason, the group had to make an extra-class effort to meet the goals.

From experience of the authors, this type of methodology where students are the center of the teaching-learning process can be perfectly implemented in engineering careers, because you already have a history of students who choose the thesis and work degree work under a similar scheme, although many casual occasions from the point of view of education, that is, it is
sufficient that the student reaches the desired result without evaluating how it gets and the added value that could be obtained during the development of his thesis.

**Bibliography**


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